A corresponding-state approach to quark-cluster matter

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(Dated: September 21, 2012)

The state of super-dense matter is essential for us to understand the nature of pulsars, but the non-perturbative quantum chromodynamics (QCD) makes it very difficult for direct calculations of the state of cold matter at realistic baryon number densities inside compact stars. The strong coupling between quarks might render quarks grouped in clusters, and at high densities but low temperature, it is conjectured that the quark-cluster matter could be in a condensed phase as self-bound objects. Nevertheless, supposing that the quark-clusters could be analogized to inert gases, we apply here the corresponding-state approach to derive the equation of state of quark-cluster matter, as was demonstrated for nuclear and neutron-star matter in 1970s. According to the calculations presented, the quark-cluster stars, which are composed of quark-cluster matter, could then have high maximum mass that is consistent with observations and, in turn, further observations of pulsar mass would also put constraints to the properties of quark-cluster matter. Moreover, the melting heat during solid-liquid phase conversion and the related astrophysical consequences are also discussed.

PACS numbers: 26.60.-c, 97.60.Gb, 21.65.Qr

I. INTRODUCTION

The state of matter above the nuclear matter density ρ_0 is still far from certainty, whereas it is essential for us to explore the nature of compact stars. At average density higher than $\sim 2\rho_0$, the quark degrees of freedom inside would not be negligible, and historically such compact stars are called quark stars [1–3]. Although cold quark matter is difficult to be created in laboratories or studied by direct QCD calculations, some efforts have been made to model quark matter and quark stars, from MIT bag model to color super-conductivity model [4]. In most of these models, quark matter is characterized by soft equation of state, because the asymptotic freedom of QCD tells us that as energy scale goes higher, the interaction between quarks becomes weaker.

However, at realistic baryon densities of compact stars, $\rho \sim 2 - 10\rho_0$, the energy scale is usually below 0.8 GeV, which is much lower than the scale where the asymptotic freedom could apply. In contrast, the non-perturbative effect should be significant, making quarks to couple strongly with each other. Quark-clustering is then conjectured to occur in cold dense matter inside compact stars, by condensation of quarks in position space due to the strong coupling [5]. The realistic quark stars could then be actually "quark-cluster stars", and solidification could be a natural result if the kinetic energy of quark-clusters is much lower than the interaction energy between them. The idea of clustering quark matter could provide us a way to understand different manifestations of pulsars [6].

How can then one model the equation of sate of quark-cluster matter? Due to the lack of both theoretical and experimental evidence, the hypothetical quark-clusters in cold dense matter has not been confirmed, and it is also difficult for us to derive the properties of quark-cluster matter by direct QCD calculations. Nonetheless, an empirical way was employed and discussed seriously in a corresponding-state approach to deduce the properties of nuclear matter in 1970s [e.g. 7, 8]. To establish a model which could be tested by observations, we adopt an empirical method here, by analogizing quark-clusters to inert gases and applying the corresponding-state approach too. A quark-cluster is assumed to be usually colorless, just like an inert atom is electric neutral. The interaction between inert gas atoms is the result of residual electromagnetic force, and similarly the interaction of quark-clusters could be seen as the result of residual strong force, both of which should be characterized by the short-distance repulsion and long-distance attraction. In this paper, we assume that the interaction between quark-clusters could be described by the same form as that between inert gas atoms, i.e. the Lennard-Jones potential, only with different parameters indicating stronger interaction and larger densities. A special kind of quark-cluster, so-called H-cluster, was studied extensively [9], and the interaction between H-clusters could be Lennard-Jones-like under the Yukawa potential with σ and ω coupling. In addition, nucleon could also be considered as special 3-quark clusters, the interaction between which is found to be Lennard-Jones-like by both experiment and modeling [10].

In fact, quark matter in Lennard-Jones model has been studied, where the equation of state is derived by summing the interaction energy of all quark-clusters [11]. The so-called corresponding state approach we demonstrated in this paper, however, is an empirical one, to derive properties of quark-cluster matter by just a comparison to the experimental data of inert gases, based on the law of corresponding states. The law of corresponding states was first proposed by de Boer [12], who found that the properties of inert gases, such as pressure and density, could be written in a reduced form.

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After reducing to dimensionless terms, the experimental data of various inter gases can be fitted in smooth curves with a single quantum parameter. If the quark-cluster matter is assumed to be similar to inert gases, the corresponding-state approach can be applicable to study the state of quark-cluster matter, without knowing its exact structure. Although interaction between quark-clusters may not be perfectly described by Lennard-Jones potential, the corresponding properties of quark-cluster matter could be in the same ranges, so our approach is to some degrees reasonable.

With the same form of interaction, we can derive the equation of state of quark- cluster matter from empirical data of inert gases, by the corresponding-state approach. The masses and radii of quark-cluster stars can then be derived and compared with observations. We find that the maximum mass of quark-cluster stars can be well above $2M_\odot$. The melting heat is also discussed, and it is shown that the solidification of newly born quark-cluster stars might explain the plateau of γ-ray bursts.

This paper is arranged as follows. A brief introduction of the law of corresponding states is given in §II. The equation of state of quark-cluster matter and the mass-radius curve of quark-cluster stars are derived in §III using the corresponding-state approach. The melting heat of solid quark-cluster stars and the related astrophysical consequences are discussed in §IV. We make conclusions and discussions in §V.

II. THE LAW OF CORRESPONDING STATES

The law of corresponding states, advocated by de Boer [12], shows that the equation of state of substances with same form of interaction can be written in a reduced and universal form. Consider a group of substances with the following properties: (1) the total potential energy due to interaction can be written as a sum of identical expressions $\varphi(r_{ik})$, each of which depends only on the distance r_{ik} between two particles i and k; (2) $\varphi(r) = \varepsilon f(r/\sigma)$, where f is a function same for all substances, and ε, σ are characteristic energy and length for different species. The macroscopic quantities, such as pressure P, volume V and temperature T, can be expressed in dimensionless terms:

$$P^* = P\sigma^3/\varepsilon \tag{1}$$

$$V^* = V/(N\sigma^3) \tag{2}$$

$$T^* = kT/\varepsilon \tag{3}$$

Another dimensionless parameter is

$$\Lambda^* = h/(\sigma\sqrt{m\varepsilon}), \tag{4}$$

corresponding to the de Broglie wavelength, which is constructed to measure the importance of quantum effects. It can be proved that the reduced equation of states expressed in dimensionless quantities is a universal relation

$$P^* = f(V^*, T^*, \Lambda^*), \tag{5}$$

which is the formulation of the law of corresponding states [12].

Despite the so-called universal equation of states is just formally written as eq.(5), a formula that is difficult to be derived theoretically for most cases, it could be used to obtain information on the equation of state of a substance which we are unfamiliar with. For determined V^* and T^*, P^* depends on the value of Λ^*, and the $P^* - \Lambda^*$ curve can be drawn using experimental data of laboratory substances. If the curve is smooth enough, the value of P^* for unfamiliar matter at such a state can be predicted provided its Λ^* is known.

For some substances described by Lennard-Jones 6-12 potential

$$\varphi(r) = \varepsilon\{\frac{4}{(r/\sigma)^{12}} - \frac{4}{(r/\sigma)^6}\}, \tag{6}$$

de Boer had determined ε and σ of noble gases and some permanent gases ($r = \sigma$ is the distance where $\varphi(r) = 0$, and ε is the depth of potential well) [12]. Then the experimental data of P^*, T^* or V^* for different substances turn out to be smooth functions of Λ^* as corresponding states. In Fig.1, experimental data of the volume V_0 at zero temperature and zero pressure, reduced to $V_0^* = V_0/(N\sigma^3)$, are plotted with Λ^* for some substances. A smooth curve

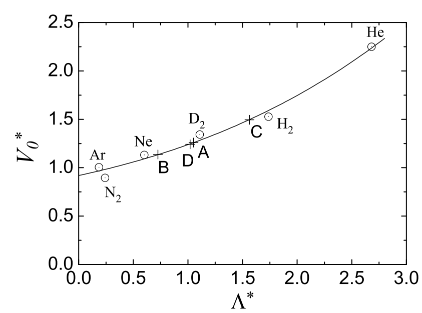

FIG. 1. Experimental data of reduced volume V_0^* and Λ^* at zero temperature and pressure for different inert gases are shown by dots [12], and the fitted curve of eq.(7) is shown by solid line. Four cases A, B, C, and D are denoted by crosses, which will be explained in §III A.

can be drawn by fitting all the points, which forms the bases of our prediction via corresponding states law, and the formula for the fitted curve is

$$V_0^* = 0.57 + 9.45 \times 10^{-5}(\Lambda^* + 6.35)^{4.44}. \tag{7}$$

Considering the property of short-distance repulsion and long-distance attraction shown by Lennard-Jones potential, we assume that the interaction between quark-clusters can also be described by this form. The distinctions between quark-cluster matter and ordinary substances should be a much deeper potential well (larger ε) and higher density (smaller σ). With the same form of interaction as that of inert gas, we could apply the

law of corresponding states to derive the properties of quark-cluster matter. If we find the quantum parameter Λ^* corresponding to quark-cluster matter, then V_0^* and other properties that vary smoothly with Λ^* can be determined by simply looking at the experimental curves of that property vs Λ^* for inert gases.

III. THE STATE OF QUARK CLUSTER MATTER

A. Parameters

To apply the law of corresponding states to quark-cluster matter, we must determine ε, σ and the mass m of each quark-cluster first. m depends on the number of quarks N_q and the mass of each quark m_0 in one cluster. We give each quark a constituent mass and assume m_0 is one-third of the nuclear mass. N_q is left as a free parameter in this paper, and we choose two cases coming from the following considerations. A dihyperon with quantum number of $\Lambda\Lambda$ (H dibaryon) was predicted to be a stale state or resonance [13], and recently Lattice QCD simulations have found evidence for its existence [14, 15]. Besides, an 18-quark cluster, i.e. quark-α, being completely symmetric in spin, color and flavor space, was also predicted to exist [16]. Consequently, we set $N_q = 6$ and $N_q = 18$ for our calculations.

As no experimental attempt has been made to get the values of ε and σ, we try to constrain their values by the surface density ρ_s of quark-cluster stars. The temperature of quark stars can be approximated to be zero, and the pressure also reaches zero at the surface of stars. Given the value of V_0^*, we can calculate the surface density ρ_s (rest-mass density). It is obvious that ρ_s can be written as

$$\rho_s = N \cdot N_q m_0 / V_0, \tag{8}$$

and comparing eq.(2) with eq.(8) we can get

$$\rho_s = N_q m_0 / (V_0^* \sigma^3). \tag{9}$$

For certain values of N_q, ε and σ, we can calculate Λ^* of quark cluster matter by eq.(4), and V_0^* can be found according to the fitted relation eq.(7) of V_0^*-Λ^* curve, then we may determine ρ_s using eq.(9). In Fig.2, pairs of ε and σ that correspond to the same surface density ρ_s are plotted respectively for $N_q = 6$ and $N_q = 18$, where values of ρ_s are chosen to be once, twice and three times of nuclear matter density ρ_0. The lines of ε and σ giving the same Λ^* with values 1, 2 and 3 are also drawn here for a further limit.

The surface density of quark stars is assumed to be in the range $1 < \rho_s/\rho_0 < 3$. Quark-clusters could condensate to form solid state like classical particles, so the quantum effects may not be large for quark-cluster matter, then Λ^* should satisfy $\Lambda^* < 2$. We select four points numbered A, B, C and D representatively to deduce the

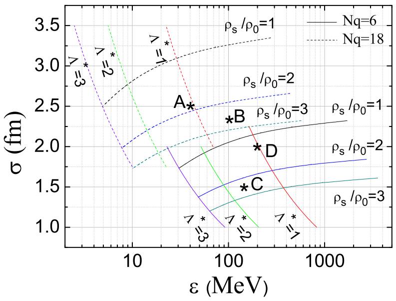

FIG. 2. Contour lines of surface density ρ_s and Λ^*, with solid lines representing $N_q = 6$ and the dashed lines representing $N_q = 18$, including $\rho_s/\rho_0 = 1, 2, 3$ and $\Lambda^* = 1, 2, 3$. Four cases A, B, C and D are denoted by stars.

equation of state for quark-cluster matter and then the mass-radius relation of quark-cluster star. The values of ε, σ and the resulting ρ_s, Λ^* at points A to D are given in Table III A. They are also plotted in Fig. 1, corresponding to four different cases for quark-cluster matter, in which the equation of state will be calculated respectively.

Table III A

	N_q	ε (MeV)	σ (fm)	Λ^*	ρ_s/ρ_0
A	18	40	2.5	1.05	1.87
B	18	100	2.3	0.72	2.72
C	6	150	1.5	1.56	2.47
D	6	200	2.0	1.02	1.23

Our choice of parameters ε and σ comes from the following consideration. The depth of potential well for nuclear matter is about 100 MeV, so it may be reasonable that $\varepsilon = \mathcal{O}(100\ \mathrm{MeV})$. The average inter-cluster distance d at the surface of quark-cluster stars is given by

$$d = [\frac{3 \times 0.16(\rho_s/\rho_0)}{N_q}]^{-\frac{1}{3}}. \tag{10}$$

With $\rho_s/\rho_0 = 2$, we get $d = 1.84$ fm for $N_q = 6$ and $d = 2.66$ fm for $N_q = 18$. Then $\sigma = \mathcal{O}(1\ \mathrm{fm})$ as it should have the same order of magnitude as d. It can be seen that the selected parameters are consistent with the above estimation.

B. The equation of state

Given ε and σ, we can deduce the state of quark-cluster matter by a corresponding-state approach, in the zero temperature case. If we know the experimental $P^* - \Lambda^*$ curve at a certain V^* and zero temperature, we can find the value of P^* corresponding to Λ^* of quark cluster.

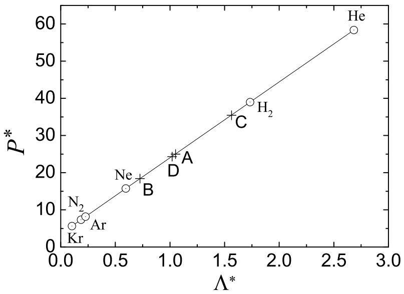

FIG. 3. Experimental data of P^* and Λ^* at zero temperature when $V^* = 0.88$ are shown by dots [17], and the fitted curve is almost a straight line (solid line). The cases A, B, C, and D are denoted by crosses.

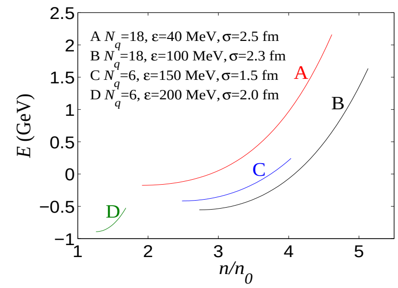

FIG. 4. The internal energy E per cluster for four groups of parameters A(red line), B(black line), C(blue line) and D(cyan line). The range of density n is from surface density n_s to the highest central density where the quark-cluster stars reaches the maximum mass.

According to eq.(1) and the number density of quark-clusters $n = 1/(V^* \sigma^3)$, the reduced quantities P^* and V^* can be converted to P and n, then we can get the pressure at a certain number density of quark-clusters. Combining this with the relation between mass density ρ (rest-mass density plus interaction energy density) and number density n of quark-cluster matter, the equation of state can be derived.

To draw the $P^* - \Lambda^*$ curve at different V^*, we need to know the relationship between P^* and V^* of some substances at zero temperature. For the lack of new data, we just use the data provided by de Boer in his subsequent article [17], where values of P^* and Λ^* were given corresponding to different values of V^* for various inert gases. Taking $V^* = 0.88$ for instance, the $P^* - \Lambda^*$ curve are shown in Fig.3. The data points are almost in linear relation, which makes our interpolation reliable. The value of Λ^* at point A is about 1.05, then we find $P^* \approx 25$ from the $P^* - \Lambda^*$ curve. The corresponding P and n to the reduced quantities P^* and V^* are $P = 1.0 \times 10^{35}$ dyn/cm^2, $n/n_0 = 2.7$ (n_0 is the number density of nucleons in nuclear matter). Thus we get $P(n = 2.7n_0) = 1.0 \times 10^{35}$ dyn/cm^2 for quark-cluster matter in case A. By taking different values of V^*, pressure P at different densities can be determined in case A. The same procedure is also applicable to the other three cases.

For each set of parameters, what we get is just a set of points in $P-n$ diagram and not an analytic equation, and then we perform the curve fitting to get an approximate formula. The $P - n$ relations derived from curve fitting are

$$P = (2.99 \times 10^{41} n^{5.63} - 1.60 \times 10^{34}) \text{ dyn/cm}^2 \quad (11)$$

$$P = (1.99 \times 10^{41} n^{5.64} - 7.63 \times 10^{34}) \text{ dyn/cm}^2 \quad (12)$$

$$P = (8.10 \times 10^{38} n^{5.24} - 1.69 \times 10^{35}) \text{ dyn/cm}^2 \quad (13)$$

$$P = (6.69 \times 10^{40} n^{5.63} - 1.63 \times 10^{35}) \text{ dyn/cm}^2 \quad (14)$$

for A, B, C and D respectively, where n is in units of clusters/fm^3. Certainly it is better to deduce the equation of state from a border range of densities, making the extrapolation to be more accurate. Nevertheless, lacking in experimental data of laboratory substances, we can only make such an approximation at this stage. It is worth mentioning that the approximation will not have much influence on the following calculations of the mass-radius curves.

According to $P = n^2 \frac{dE}{dn}$, where E is the internal energy per cluster, we can get

$$E(n) - E(n_s) = \int_{n_s}^{n} \frac{P(n)}{n^2} dn, \quad (15)$$

where n_s is the number density of quark-clusters on the surface of stars. We may determine the value of $E(n_s)$ from a corresponding-state point of view, and then the relation between E and n can be derived by the above integral. Similar to V_0^*, $U_0^* = U_0/N\varepsilon$ can be approximated as a smooth function of Λ^*, where U_0 is the internal energy at zero temperature and zero pressure. From the data of laboratory substances [12], we derive a fitted formula for U_0^*,

$$U_0^* = -8.72 + 4.91\Lambda^* - 0.71\Lambda^{*2}, \quad (16)$$

and $E(n_s)$ is thus

$$E(n_s) = U_0/N = U_0^* \varepsilon. \quad (17)$$

As both $P(n)$ and $E(n_s)$ are known, it is able to calculate $E(n)$ from eq.(15). The results are plotted in Fig.4, for four groups of parameters A to D, and we can see that the internal energy can be comparable to rest-mass energy at some densities.

The mass density ρ consists of rest-mass density and energy density,

$$\rho = n(N_q m_0 + E/c^2), \quad (18)$$

then the equation of state for quark-cluster matter can be derived by combining $P - n$ relation and eq.(18), and we show the results in Fig.5, for the four groups of parameters A to D.

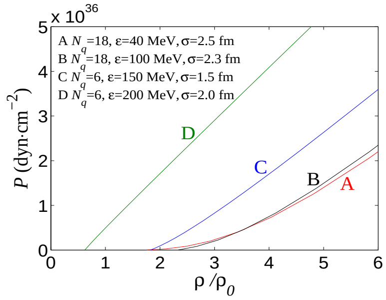

FIG. 5. Equations of states for the same four groups of parameters as in Fig. 4.

C. Mass-radius relation

Considering perfect fluid case and the general relativity, the hydrostatic equilibrium in spherically symmetry is described by Tolman-Oppenheimer-Volkoff equation,

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2}\frac{(1+\frac{P}{\rho c^2})(1+\frac{4\pi r^3 P}{m(r)c^2})}{1-\frac{2Gm(r)}{rc^2}}, \tag{19}$$

where $m(r) = \int_0^r \rho \cdot 4\pi r'^2 dr'$. In the above discussions, we have got the equations of state, from which we can make a further calculation of the mass-radius and mass-central density (rest-mass density) relations for quark-cluster stars. The results are shown in Fig.6, for the four groups of parameters A to D, and we can see that the maximum masses are higher than three times the solar mass $M_\odot$ for all the selected groups of parameters. As a comparison, we also plot the mass-radius curves for homogeneous spheres with the same central density corresponding to each of the four cases. This shows that the gravity cannot be negligible only when the stars is near the maximum mass, which could be the result of the strong self-bounding of quark-cluster stars.

Conventional quark matter is characterized by soft equation of state, and the emerge of quark matter inside compact stars is usually thought to be a reason for lowering their maximum mass. The quark-cluster matter, however, could have stiff equation of state due to the strong coupling. Although the corresponding-state approach is just a phenomenological and empirical method, we could still apply it to study the state of quark-cluster matter and then understand the observations of pulsar-like compact stars. The observed high-mass pulsar PSR J1614-2230 with mass $1.97 \pm 0.04 M_\odot$ [18] has received a lot of attention, and we can see that the quark-cluster stars in our present model could be consistent with this observation. Moreover, our model of quark-cluster stars could not be ruled out even if the mass of the pulsar J1748-2021B ($2.74 M_\odot$) in a galactic cluster is confirmed in the future.

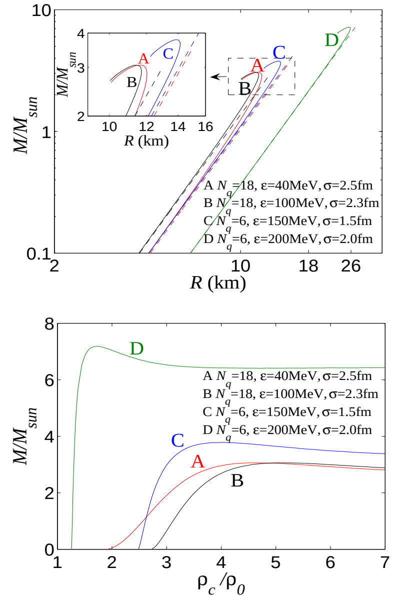

FIG. 6. The mass-radius and mass-central density (rest-mass density) curves, and different parameters are distinguished by their colors as the same in Fig 4, and the corresponding dash lines represent $M = \rho_s \cdot 4\pi R^3/3$.

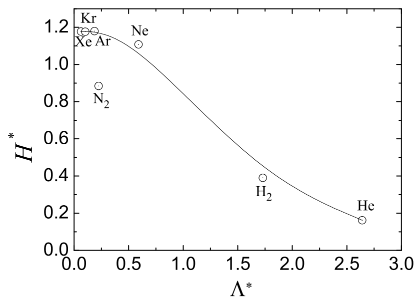

FIG. 7. Data points: experimental data of the reduced melting heat $H^* = H/\varepsilon$. Solid line: fitted curve of eq.(20).

IV. MELTING HEAT

If the kinetic energy of quark clusters is much lower than the inter-cluster potential energy, they may form a solid state. We will estimate the latent heat of phase transition of quark-cluster stars from liquid to solid state by the corresponding-state approach.

We calculate the ratio of melting heat per particle H and ε for some ordinary substances [19], and find that there is also a good relation between $H^* = H/\varepsilon$ and Λ^*, as shown in Fig.7. The fitted formula for H^* and Λ^* is

$$H^* = 1.18e^{-((\Lambda^*-0.12)/1.60)^2}. \tag{20}$$

For given N_q, ε and σ, we can determine Λ^* first and then get the value of H^* from eq.(20), thus the melting heat $H = H^*\varepsilon$ can be derived.

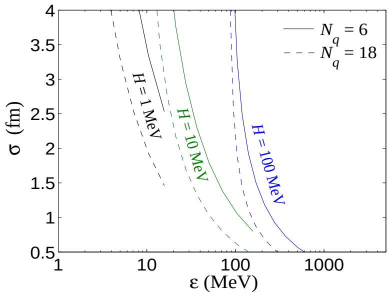

FIG. 8. ε and σ which determine the same melting heat of each cluster. H is chosen to be 1 (black line), 10 (cyan line) and 100 MeV (blue line), in two cases $N_q = 6$ (solid line) and $N_q = 18$ (dashed line).

In Fig.8 pairs of ε and σ which determine the same melting heat are plotted, where values of H are chosen to be 1, 10 and 100 MeV, in two cases $N_q = 6$ and $N_q = 18$. The solidification of quark-cluster stars has been suggested to be relevant to the plateau of γ-ray burst[20], and it is found that if the energy released by each quark-cluster in the liquid to solid phase transition is larger than 1 MeV, the total released energy could produce the plateau. We can see that under a wide range of parameters in our model, the latent heat could be sufficient for this way of understanding the plateau of γ-ray burst.

V. CONCLUSIONS AND DISCUSSIONS

In cold quark matter at realistic baryon densities of pulsar-like compact stars, the interaction between quarks would be so strong that they could condensate in position space, forming quark-clusters, and the stars are then called quark-cluster stars if the dominant component inside is quark-clusters. We propose that the interaction between quark-clusters is like that between inert gas atoms described by the Lennard-Jones potential, and apply the corresponding-state approach to derive the equation of state. As a phenomenological and empirical method, the corresponding-state approach can avoid detailed assumptions of quark-cluster matter as well as computation of the many-body effects, and we only need to concern about differences between substances. Along with of these advantages, there are large uncertainty in our results, coming from the Lennard-Jones approximation and lack of experimental data source. Even so, the corresponding-state approach could give us qualitative information about the properties of quark-cluster matter, while the exact approach under QCD calculations seems to be impossible due to the significant non-perturbative effect. Summarily, our two-parameter (ε and σ) empirical approach make it possible to establish a model which could be tested by observations.

The equation of state we have derived by the corresponding-state approach could be stiff enough to make a star stable even if its mass is higher than $2M_\odot$, under reasonable parameters. This result is consistent with the recent observation of a high-mass pulsar, thus the emergence of such kind of exotic matter, "quark-cluster matter", could not be ruled out. The observations of pulsars with higher mass, e.g. $> 3M_\odot$, would even be a support to our quark-cluster star model, and give further constraints to the parameters. Moreover, the latent heat released by the solidification of newly born quark-cluster stars could help us to understand the formation of the plateau of γ-ray burst.

Whether quark-cluster matter could exist at supranuclear densities, and what quark-clusters are composed of, as well as how to describe their interaction are still open questions. On the other hand, the nature of pulsar-like compact stars is still uncertain. These are all essentially related to the non-perturbative QCD, and we hope that future astrophysical observations, complementary to the territorial experiments, could give us hints to all of these questions.

VI. ACKNOWLEDGEMENT

We would like to thank useful discussions at our pulsar group of PKU. This work is supported by the National Natural Science Foundation of China (Grant Nos. 10935001, 10973002), the National Basic Research Program of China (Grant No. 2012CB821800) and the John Templeton Foundation.

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